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Central charge for 2D gravity on AdS_2 and AdS_2/CFT_1 correspondence

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Abstract: We study 2D Maxwell-dilaton gravity on AdS_2. We distinguish two distinctive cases depending on whether the AdS_2 solution can be lifted to an AdS_3 geometry. In both cases, in order to get a consistent boundary condition we need to work with a twisted energy momentum tensor which has non-zero central charge. With this central charge and the explicit form of the twisted Virasoro generators we compute the entropy of the system using the Cardy formula. The entropy is found to be the same as that obtained from gravity calculations for a specific value of the level of the U(1) current. The agreement is an indication of AdS_2/CFT_1 correspondence.

Keywords: Black Holes in String Theory, AdS-CFT Correspondence, Black Holes.

Contents

1. Introduction

In 3D gravity the group of diffeomorphisms which preserves the condition that the metric be asymptotically AdS_3 is two copies of the Virasoro algebra with the central charge [1]

$$c = \frac{3l_3}{2G_3}, \tag{1.1}$$

where l_3 is the AdS_3 radius and G_3 is the three dimensional Newton's constant.

This fact has been used to compute the entropy of three dimensional black holes. It has been shown [2] that one may use the Cardy formula with the above central charge and count the boundary degrees of freedom which agrees with the entropy of the black hole in the bulk. Therefore the symmetry is enough to find the entropy without knowing about the details of the dynamics.

Of course we now understand the reason behind this precise agreement and it is due to the AdS/CFT correspondence [3]. In three dimensions the lesson we have learned from the AdS/CFT correspondence is that "any consistent quantum gravity on an asymptotically AdS_3 spacetime is a 2D CFT living on the boundary of the AdS_3".

Although $\text{AdS}_{d+1}/\text{CFT}_d$ have been understood for $d \geq 2$ mainly due to explicit examples, little has been known for the case of $d = 1$ (see however [4−9]). The lack of our knowledge of the holographic dual of AdS_2 is due to the special features of AdS_2 spacetime. First of all it has two boundaries. Secondly, so far we do not have a concrete example of an $\text{AdS}_2/\text{CFT}_1$ in the context of string theory where we could identify both sides of the duality.

On the other hand the quantum gravity on AdS_2 geometry is important on its own right. Indeed the AdS_2 geometry is the factor which appears in the near horizon geometry of the extremal black holes in any dimension. Therefore understanding gravity on AdS_2 might ultimately help us understand the origin of the black hole entropy in other dimensions.

To explore the AdS$_2$/CFT$_1$ correspondence one may utilize the experience of the AdS_3 case; namely, one could try to understand the asymptotic symmetry of AdS_2. In fact this has been done in several papers including [4 − 7]. In particular it has been shown [4] that the asymptotic symmetry group of an asymptotically AdS_2 geometry is one copy of the Virasoro algebra. Recently it has also been shown [10] that exactly the same argument as that for AdS_3 [1] can be made for quantum gravity with a U(1) gauge field on AdS_2 leading to the following central charge[1]

$$c = 12kG^2Q^2l^4, \tag{1.2}$$

where l is the radius of AdS_2, Q is the electric charge, k is the level of the current which generates the U(1) and G is the two dimensional Newton's constant.

The aim of this article is to further study 2D Maxwell-dilaton gravity on AdS_2 background. In particular we would like to study the entropy of the corresponding solution and to see to what extent the information of the theory is encoded in a CFT whose central charge is given by (1.2). Our strategy is the 2D analog of [2]. Namely we will reproduce the black hole entropy obtained from gravity by making use of the Cardy formula with the central charge given by the asymptotic conformal diffeomorphism of the 2D theory. As observed in [10] in order to have a consistent boundary condition for gravity on AdS_2 coupled to gauge field, the energy momentum tensor has to be twisted. Having a non-zero central charge plus the fact that we can consistently reproduce the entropy from this central charge provides an indication of the AdS_2/CFT_1 correspondence showing that the corresponding holographic dual would be a chiral half of a 2D CFT (see also [10, 11]).

In the course of studying 2D Maxwell-dilaton gravity on AdS_2 we will encounter two distinctive cases depending on whether the corresponding AdS_2 solution can be lifted up into AdS_3 solution. These two models are given by the actions

$$\begin{aligned} S_1 &= \frac{1}{8G}\int d^2x\sqrt{-g}\left(e^{\phi}\left(R+\frac{8}{l^2}\right)-\frac{l^2}{4}F^2\right), \\ S_2 &= \frac{1}{8G}\int d^2x\sqrt{-g}\,e^{\phi}\left(R+\frac{2}{l^2}-\frac{l^2}{4}e^{2\phi}F^2\right). \end{aligned} \tag{1.3}$$

Both actions admit an AdS_2 vacuum solution. We note, however, that although in the second case the AdS_2 solution can be lifted up to three dimensional AdS_3 solution, in the first one it cannot. Our main conclusion is that for both cases we can reproduce the black hole entropy using the central charge of the twisted energy momentum tensor. Moreover we can show that in the first case approaching the horizon a CFT emerges which has the same central charge as that obtained by using the asymptotic symmetry. Therefore in this case there may be a correspondence between the two CFT's; one at infinity and the other at the horizon.

The paper is organized as follows. In section two we will study 2D Maxwell-dilaton gravity on AdS_2 based on the first action in (1.3). We will show that the central charge of the twisted energy momentum can be used to compute the entropy using the Cardy

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[1] In order to compare the result with that in [10] one needs to use a unit in which $G = \frac{1}{4}$.

formula. The entropy is the same as the black hole entropy obtained from the gravity side for a specific value of k. Therefore the consistency of the results leads us to fix the level of U(1) current too. We will also study the model in terms of the near horizon modes where we show that at near horizon we get a CFT whose central charge is the same at that obtained from asymptotic symmetry. In section three we will do similar computations for the model based on the second action in (1.3). We will see that using the asymptotic symmetry one can read off the central charge of the twisted energy momentum tensor which can then be used to reproduce the black hole entropy correctly. The last section is devoted to the conclusions and discussions.

2. Type I 2D gravity

2.1 Central charge and entropy

In this section we study the 2D Maxwell-dilaton gravity based on the action

$$S_1 = \frac{1}{8G}\int d^2x\sqrt{-g}\left(e^{\phi}\left(R+\frac{8}{l^2}\right)-\frac{l^2}{4}F^2\right). \tag{2.1}$$

This model has recently been considered in [10] where the central charge of the CFT associated with the asymptotic symmetry of its AdS_2 solution was calculated. The authors of [10] noticed that the potential of the U(1) gauge field is singular at the boundary and therefore the boundary condition is not respected under a conformal diffeomorphism. The resolution was to accompany the conformal transformation with a special U(1) gauge transformation, generated by a current $j_{\pm}$, leading to the twisted energy momentum tensor [10]

$$\tilde{T}_{\pm\pm} = T_{\pm\pm} \pm GQl^2\partial_{\pm}j_{\pm}, \tag{2.2}$$

where T is the energy momentum tensor corresponding to the original conformal transformation with $c = 0$.

Since the AdS_2 solution carries an entropy, it is natural to pose the question of whether this entropy can be reproduced using the Cardy formula with central charge given by (1.2). In other words we should be able to compute the *statistical* entropy by evaluating the eigenvalue of $\tilde{L}_0$ coming from energy momentum (2.2) and then use the Cardy formula with the central charge (1.2). The resultant entropy should be compared with the entropy of the AdS_2 solution.

Therefore it is important to first compute the entropy of the model using the gravity solution which we do utilizing the entropy function formalism [12], which only needs the information of the near horizon geometry. Since the geometry is AdS_2, we start from an ansatz respecting the SO(2, 1) isometry of the AdS_2 solution,

$$ds^2 = v\left(-r^2dt^2+\frac{dr^2}{r^2}\right), \qquad e^{\phi}=\eta, \qquad F_{rt}=\frac{e}{l^2}, \tag{2.3}$$

where v, η and e are constant to be determined by the equations of motion.

To proceed we need to evaluate the entropy function,

$$\mathcal{E} = 2\pi(Qe - f(e, v, \eta)), \tag{2.4}$$

where

$$f = \frac{v}{8G}\left[\eta\left(-\frac{2}{v}+\frac{8}{l^2}\right)+\frac{e^2}{2v^2l^2}\right] \tag{2.5}$$

is the Lagrangian density evaluated for the ansatz. Extremizing the entropy function with respect to parameters v, η and e we get

$$ds^2 = \frac{l^2}{4}\left(-r^2dt^2+\frac{dr^2}{r^2}\right), \qquad e^{\phi} = 4G^2Q^2l^4, \qquad F_{rt} = 2GQl^2. \tag{2.6}$$

The entropy is also given by

$$S_{BH} = 2\pi GQ^2l^4 \tag{2.7}$$

which can be recast in the suggestive form

$$S_{BH} = 2\pi\sqrt{\frac{1}{6}(12GQ^2l^4)\left(\frac{12GQ^2l^4}{24}\right)}, \tag{2.8}$$

reminiscent of the Cardy formula

$$S = 2\pi\sqrt{\left(\frac{c}{6}-4\Delta_0\right)\left(\Delta-\frac{c}{24}\right)}, \tag{2.9}$$

where Δ is the eigenvalue of L_0 and Δ_0 its the lowest eigenvalue. With this observation, the task is to see whether the CFT defined by the twisted energy momentum tensor (2.2) and central charge (1.2) can in fact reproduce the above entropy.

To proceed we note that the twisted energy momentum tensor (2.2) satisfies the following commutator relation [10]

$$\begin{aligned}[\tilde{T}_{--}(t^-),\tilde{T}_{--}(s^-)] = &-4\pi\partial_-\delta(t^- - s^-)\tilde{T}_{--}(s^-) + 2\pi\delta(t^- - s^-)\partial_-\tilde{T}_{--}(s^-)\\ &+2\pi kG^2Q^2l^4\partial_-^3\delta(t^- - s^-),\end{aligned} \tag{2.10}$$

leading to the central charge of $c = 12kG^2Q^2l^4$. k is the coefficient of the Schwinger term in the U(1) current algebra

$$[j_-(t^-), j_-(s^-)] = -2\pi k\partial_-\delta(t^- - s^-). \tag{2.11}$$

These expressions together with the definition of the twisted energy momentum tensor show that upon mode expanding the energy momentum tensor, the Virasoro generators become

$$\tilde{L}_n = L_n - GQl^2U_n, \tag{2.12}$$

which satisfy a Virasoro algebra as follows

$$[\tilde{L}_n, \tilde{L}_m] = (n-m)\tilde{L}_{n+m} + \frac{12kG^2Q^2l^4}{12}(n^3-n)\delta_{n+m,0}. \tag{2.13}$$

Here U_n are the coefficients of the mode expansion of $\partial_- j_-$.

We now want to evaluate the eigenvalues of L_0 and U_0. This may be done by using the semiclassical method in which Δ is obtained from the integral of the energy momentum tensor T evaluated at the AdS solution which turns out to be zero. On the other hand to find U_0 note that the non-zero value of the divergence of the U(1) current, $\partial_- j_-$, which gives an important contribution to the energy momentum tensor, is the anomaly related to the Schwinger term in (2.11). As it has already been pointed out in [10] the situation is very similar to the Schwinger model [13] where, using the fermionic formulation, one gets the central term in the commutator, and the anomaly in the U(1) current, due to the one loop digram contributions. This observation can be used to evaluate the divergence of the current. Indeed using the standard anomaly and the Schwinger term calculations one finds (see for example [14, 15])

$$\partial_- j_- = \frac{k}{4}\,\epsilon^{\mu\nu}F_{\mu\nu} = -kGQl^2. \tag{2.14}$$

So that $\tilde{\Delta} = kG^2Q^2l^4$. Therefore with the assumption $\tilde{\Delta}_0 = 0$ and using the central charge $c = 12kG^2Q^2l^4$ we arrive at

$$S = 2\pi GQ^2l^4\,(kG). \tag{2.15}$$

Comparing the Cardy entropy with the black hole entropy (2.7) one has to set $k = \frac{1}{G}$. Thus to get a consistent result the level of U(1) is not a free parameter. Of course we are familiar with such a phenomena; namely requiring to get correct black hole entropy may put a constraint on the level of affine algebra in the dual CFT (for example see [16 − 21]). So the central charge and $\tilde{\Delta}$ of the CFT read

$$c = 12GQ^2l^4, \qquad \tilde{\Delta} = GQ^2l^4. \tag{2.16}$$

We would like to identify this central charge as the central charge of a CFT whose global SL$(2,R)$ symmetry is the isometry of the AdS_2 geometry. With the above particular value of k, the CFT descriptions is consistent with gravity result. This is a strong indication in favor of the AdS_2/CFT_1 correspondence.

2.2 Near horizon modes

There is an alternative CFT living in the near horizon region of the black hole describing its entropy. In this approach one may identify the entropy as the number of states at the horizon. To do this we follow [22] and make a change of variables in the action S_1,

$$\frac{e^{\phi}}{2G} = \Phi^2 = q\Phi_0\varphi, \qquad g_{\mu\nu} \to e^{\frac{2}{q\Phi_0}\varphi}g_{\mu\nu}, \tag{2.17}$$

where Φ_0 is the value of Φ at horizon, $\Phi_0^2 = 2GQ^2l^4$, and q is a free parameter and get

$$S_1 = -\int d^2x\sqrt{g}\left(-\frac{1}{4}q\Phi_0\varphi R + \frac{1}{2}(\nabla\varphi)^2 - \frac{2q\Phi_0}{l^2}\varphi e^{\frac{2}{q\Phi_0}\varphi} + \frac{l^2}{32G}e^{-\frac{2}{q\Phi_0}\varphi}F^2\right). \tag{2.18}$$

Then integrating out the gauge field, using the Maxwell equations, leads to an effective potential for the scalar field φ

$$S_1 = -\int d^2x\sqrt{g}\left(-\frac{1}{4}q\Phi_0\varphi R + \frac{1}{2}(\nabla\varphi)^2 + V(\varphi)\right). \tag{2.19}$$

This is the action considered in [22] where it was shown that in the near horizon limit ($r \to 0$) the energy momentum tensor of the theory becomes traceless leading to a CFT with a specific central charge.

To be precise the author of [22] starts from a d-dimensional spherically symmetric black hole and decomposes the metric into two parts

$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + h_{ij}(r)dx^i dx^j. \tag{2.20}$$

Dimensionally reducing along the x^i's he gets, after some manipulations, the above two dimensional action.

Although the considerations of [22] are for a non-extremal black hole where the leading behavior of g in the near horizon limit is $g(r) \sim (r - r_h)$ with r_h being the radius of the horizon, the procedure works for the extremal case as well where $g(r) = vr^2$. The only difference is that in the non-extremal case the trace of energy momentum tensor vanishes exponentially when the horizon is approached while in our case the approach is power law.

Using the change of variable $z = \frac{1}{r}$ (note that in this case horizon is at $z \to \infty$), the components of the energy momentum tensor of the action (2.19) read

$$\begin{aligned}
T_{00} &= \frac{1}{4}((\partial_t\varphi)^2 + (\partial_z\varphi)^2) - \frac{q\Phi_0}{4}(\partial_z^2\varphi - \frac{v}{2z}\partial_z\phi) + \frac{v}{z^2}V(\varphi),\\
T_{zz} &= \frac{1}{4}((\partial_t\varphi)^2 + (\partial_z\varphi)^2) + \frac{q\Phi_0}{4}\Big(- \partial_t^2\varphi + \frac{v}{2z}\partial_z\phi\Big) - \frac{v}{z^2}V(\varphi),\\
T_{0z} &= \frac{1}{2}\partial_t\partial_z\phi - \frac{q\Phi_0}{4}\Big(\partial_z\partial_t\varphi - \frac{v}{2z}\partial_t\varphi\Big).
\end{aligned} \tag{2.21}$$

Using the light-cone coordinates $z_\pm = t \pm z$ the non-zero components of the energy momentum tensor are given by

$$T_{\pm\pm} = (\partial_\pm\varphi)^2 \mp \frac{1}{2}q\Phi_0\partial_\pm^2\varphi, \tag{2.22}$$

which has the same structure as (2.2). Essentially it has the form of a scalar field in the presence of a non-zero background charge. If we define the Virasoro algebra as the Fourier coefficients in the expansion of the above energy momentum tensor,

$$L_n = \frac{L}{2\pi}\int_{-L/2}^{L/2} dz\; e^{2\pi i n z/L} T_{++}, \tag{2.23}$$

then it is easy to see that the shifted Virasoro generators

$$\tilde{L}_n = L_n + \frac{c}{24}\delta_{n,0}, \tag{2.24}$$

satisfy a Virasoro algebra with central charge $c = 3\pi q^2\Phi_0^2$ [23]. Here we compactified z coordinate on a circle of circumference L. Finally we would like to send L to infinity.

The aim is to evaluate L_0 for our background. It is, however, tricky as the scalar field is constant and naively leads to zero L_0. Nevertheless note that in the vicinity of the horizon the equations of motion allow a more general zero mode configuration $\varphi = 2\Phi_0/qL\; z$; in our notation the horizon is at $z = L/2$. Plugging this expression in the definition of the energy momentum tensor and using the definition of L_n, (2.23), one arrives at

$$c = 6\pi q^2 G Q^2 l^4, \qquad \tilde{\Delta} - \frac{c}{24} = \frac{GQ^2 l^4}{\pi q^2}. \tag{2.25}$$

For $q^2 = \frac{2}{\pi}$ these are exactly the same results we obtained using the asymptotic symmetry of the model.

Observe that the effect of the U(1) gauge transformation at asymptotic boundary condition reflects, upon integrating out the gauge field, a non-zero background charge in the near horizon limit description of the theory. Therefore in this case twisting at infinity has the same effect as having a non-zero background charge at the horizon. We conclude that there may be a one to one correspondence between these two CFT's, one at infinity and the other at the horizon (for discussions on these two CFT's see for example [24]).

3. Type II 2D gravity

In this section we consider 2D Maxwell-dilaton gravity based on the following action

$$S_2 = \frac{1}{8G}\int d^2x\sqrt{-g}\, e^{\phi}\left(R + \frac{2}{l^2} - \frac{l^2}{4}e^{2\phi}F^2\right), \tag{3.1}$$

which can actually be obtained from the 3D pure gravity with cosmological constant by reducing to two dimensions along an S^1. This action has been used to study entropy of extremal black hole in the presence of higher order corrections (see for example [25]).

We will redo our previous section's computations for the action (3.1). To start with note that this action also has an AdS_2 solution. Following our discussion in the previous section we start with the following ansatz

$$ds^2 = v\left(-r^2dt^2 + \frac{dr^2}{r^2}\right), \qquad e^{\phi} = \eta, \qquad F_{rt} = \frac{e}{l^2}, \tag{3.2}$$

where v, η and e are constants to be determined by the equations of motion and use the entropy function formalism to find

$$ds^2 = \frac{l^2}{4}\bigg(-r^2dt^2 + \frac{dr^2}{r^2}\bigg), \qquad e^{\phi} = \sqrt{4GQl^2}, \qquad F_{tr} = \sqrt{\frac{1}{16GQl^2}} \tag{3.3}$$

with the entropy,

$$S_{BH} = 2\pi\sqrt{\frac{Ql^2}{4G}}. \tag{3.4}$$

Following our discussions in the previous section the goal is to reproduce this entropy using the number of states of a CFT which can be defined by asymptotic symmetry of the AdS_2 solution.

Note that the above solution, unlike the solution in the previous section, has the underlying symmetry of AdS_3. In other words one may lift the two dimensional solution to three dimensions as follows

$$ds^2_{(3)} = ds^2_{(2)} + l^2 e^{2\phi}(dz + A_\mu dx^\mu)^2, \tag{3.5}$$

which for our solution, defining $y = \sqrt{16GQl^2}z$, becomes

$$ds^2_{(3)} = \frac{l^2}{4}\left(dy^2 + 2r^2 dydt + \frac{dr^2}{r^2}\right), \tag{3.6}$$

clearly the AdS_3 written in the S^1 fibered over AdS_2 coordinates. So we get the AdS_3 with an identification and the solutions is lifted to the extremal BTZ black hole.

Taking into account the isometry of AdS_3 which is $\mathrm{SL}(2,R)_R \times \mathrm{SL}(2,R)_L$, our two dimensional solution can be thought of as an $\mathrm{SL}(2,R)_L$ quotient of AdS_3 [26]. Therefore we are left with just the $\mathrm{SL}(2,R)_R$ symmetry of AdS_3 which under the reduction reduces to the $\mathrm{SL}(2,R)$ isometry of AdS_2 plus a gauge transformation [4]. Following the arguments of [10] one can see that the conformal diffeomorphism in two dimensions will not respect the boundary conditions of the gauge field, necessitating an extra $\mathrm{U}(1)$ gauge transformation leading to a twisted energy momentum tensor [4]

$$\tilde{T}_{\pm\pm} = T_{\pm\pm} \pm \partial_\pm j_\pm, \tag{3.7}$$

where $j_\pm$ is the appropriately normalized current associated with the Kaluza-Klein $\mathrm{U}(1)$ gauge symmetry, and T is the AdS_2 energy momentum tensor with zero central charge.

The central charge of the twisted quantum theory crucially depends on the correct normalization of the $\mathrm{U}(1)$ current. In our case there is a short cut in calculating the central charge. The Virasoro generators $\tilde{L}_n$ corresponding to the twisted energy momentum tensor $\tilde{T}$, are equal to the right handed modes $L_n^{(3)}$ of the energy momentum tensor of AdS_3 corresponding to its $\mathrm{SL}(2,R)_R$ isometry, $L_n^{(3)} = \tilde{L}_n$ [4]. As

$$[L_n^{(3)}, L_m^{(3)}] = (n-m)L_{n+m}^{(3)} + \frac{c^{(3)}}{12}(n^3 - n)\delta_{n+m,0}, \tag{3.8}$$

with $c^{(3)} = 3l/2G_3$, we get

$$[\tilde{L}_n, \tilde{L}_m] = (n-m)\tilde{L}_{n+m} + \frac{1}{12}\left(\frac{3}{2G}\right)(n^3 - n)\delta_{n+m,0}, \tag{3.9}$$

where $G_3 = lG$. This gives the central charge of the twisted energy momentum tensor,

$$c = \frac{3}{2G}. \tag{3.10}$$

The entropy can now be computed from the Cardy formula following the procedure of the section two and turns out to be consistent with the gravity computations for $k = \frac{1}{4G}$ where $\tilde{\Delta} = Ql^2$. This again is a direct indication in favor of AdS_2/CFT_1 correspondence.

4. Conclusions and discussions

In this paper we have studied 2D Maxwell-dilaton gravity on AdS_2 geometry. We have seen that there are two distinctive models of 2D Maxwell-dilaton gravity defined by actions S_1 and S_2 in (1.3).

Both actions admit an AdS_2 vacuum solution. In both cases to maintain the consistent boundary conditions of the fields in the theory the conformal diffeomorphism in the boundary at infinity has to be accompanied by a special U(1) gauge transformation. The theories are well-described by the twisted energy momentum tensor defined by

$$\tilde{T}_{\pm\pm} = T_{\pm\pm} \pm A\partial_{\pm} j_{\pm}, \tag{4.1}$$

where A is a constant depending on the normalization of the U(1) current $j_{\pm}$. Although the Virasoro algebra of T has zero central charge, as expected from 2D quantum gravity, the twisted energy momentum tensor leads, after fixing the constant A correctly, to the non-zero central charge given by

$$c = 12GQ^2l^4, \qquad c = \frac{3}{2G} \tag{4.2}$$

for the models based on S_1 and S_2, respectively; which can then be used in the Cardy formula to compute the number of states of the CFT. It was then observed that the resultant entropy is equal to the entropy of the AdS_2 solution obtained from gravity calculations. This precise agreement can be thought of as a strong indication of AdS_2/CFT_1 correspondence. In particular it might be a sign that the holographic dual of gravity on AdS_2 is a chiral half 2D CFT. In fact one may go further and claim that any consistent 2D quantum gravity on an AdS_2 is dual to a chiral half 2D CFT generalizing the situation of AdS_3.

Although these models show certain similarities, they have significant differences. For example the central charge of the first model depends on the detail of the solution considered, while in the second case it only depends on the two dimensional Newton's constant. In the first model even though we have a gauge field, the solution can not be lifted to an AdS_3 geometry. In fact there are indications that the model may be obtained from a higher dimensional extremal black hole. Actually following [22] we started from a four dimensional extremal black hole with near horizon geometry $AdS_2 \times M_2$ and reducing along the M_2 manifold we found an effective Maxwell-dilaton gravity on AdS_2. In this approach the entropy is associated with the near horizon modes where a new CFT emerges. We have seen that the emergent CFT has the same central charge as that obtained using asymptotic symmetry. Moreover we have observed that the energy momentum tensor has the form of a twisted energy momentum tenor where the extra twist term can be interpreted as a non-zero background field (or Liouville theory). Indeed as far as the energy momentum tensor, central charge and entropy are concerned there is a correspondence between the two CFT's; one at infinity and the other at the horizon. This observation requires further study.

On the other hand the AdS_2 solution of the second case was lifted to an AdS_3 solution, exhibiting a direct connection between the two and three dimensional theories. In particular we were able to relate the central charge of the twisted energy momentum tensor to the Brown and Henneaux-like central charge.

It is tempting to ask whether this connection can lead to an understanding of certain aspects of one theory form the other. In particular it has been recently observed that by adding a Chern-Simons term to the three dimensional gravity, a chiral gravity is obtained [27]. It is then interesting to study the resultant chiral gravity in the 2D theory.

Actually a two dimensional Chern-Simons term can be added to our theory. In our notation the Chern-Simons action is given by [28, 29]

$$S_{cs} = \frac{1}{32G\mu}\int d^2x\, e^{2\phi}\left(lR\epsilon^{\mu\nu}F_{\mu\nu} + l^3 e^{2\phi}\epsilon^{\mu\nu}F_{\mu\rho}F^{\rho\delta}F_{\delta\nu}\right). \tag{4.3}$$

It is easy to show that a model based on $S_2 + S_{cs}$ still has an AdS_2 solution. The corresponding central charge can be computed and is seen to get corrected. The correction of the central charge depends on the sign of the electric field e. Using entropy function formalism, we get

$$\begin{aligned} c &= \frac{3}{2G}\left(1 - \frac{1}{l\mu}\right), \qquad \text{for} \quad e > 0 \\ c &= \frac{3}{2G}\left(1 + \frac{1}{l\mu}\right), \qquad \text{for} \quad e < 0. \end{aligned} \tag{4.4}$$

A chiral gravity in two dimensions, may be also defined in the same way as in three dimensions. We note, however, that a priori there is no reason from the two dimensional point of view to force the relation $\mu l = 1$. Nevertheless since the theory is related to the three dimensional theory, one would expect to find some tachyonic modes leading to a particular value for μ. Work on this question is in progress.

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